# MATH 464 (THEORY OF PROBABILITY) HOMEWORK

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(1) Let $A$, $B$, $C$ be events. Write each of the following events in terms of $A, B, C$ using intersections, unions, and complements:
  (a) All of the vents $A, B, C$ occur.
  (b) $A$ occurs and at least one of $B$ or $C$ occurs.
  (c) Exactly one of $A, B, C$ occurs.
  (d) At least one of $A, B, C$ occurs.

(2) Let $p_1, p_2, \ldots, p_N$ be non-negative numbers such that $p_1 + p_2 + \ldots + p_N = 1$ and let $\Omega = \{\omega_1, \omega_2, \ldots, \omega_N\}$, with $\mathcal{F}$ the power set of $\Omega$. Show that the function $\mathbb{Q}$ given by

$$\mathbb{Q}(A) = \sum_{j:\ \omega_j \in A} p_j \quad \text{for } A \in \mathcal{F}$$

is a probability measure on $(\Omega, \mathcal{F})$.

(3) Given two events $A$ and $B$ with $\mathbb{P}(A) = 0.4$ and $\mathbb{P}(B) = 0.7$. What is the maximum and minimum possible values for $\mathbb{P}(A \cap B)$?

(4) Suppose $\mathbb{P}(A) = \frac{1}{3}$, $\mathbb{P}(A^c \cap B^c) = \frac{1}{2}$, and $\mathbb{P}(A \cap B) = \frac{1}{4}$. Find $\mathbb{P}(B)$.

(5) Show that

$$\mathbb{P}(A \cup B \cup C) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - \mathbb{P}(A \cap B) - \mathbb{P}(A \cap C) - \mathbb{P}(B \cap C) + \mathbb{P}(A \cap B \cap C)$$

(6) Prove that (use induction)

$$\mathbb{P}\left(\bigcup_{j=1}^{n} A_j\right) \le \sum_{j=1}^{n} \mathbb{P}(A_j).$$

(7) Prove that

$$\mathbb{P}\left(\bigcap_{j=1}^{n} A_j\right) \ge 1 - n + \sum_{j=1}^{n} \mathbb{P}(A_j).$$

(8) Let $A$ and $B$ be two events. Show that if $\mathbb{P}(A) = \mathbb{P}(B) = 1$, then $\mathbb{P}(A \cap B) = 1$.

(9) Consider an unfair coin which has probability $1/3$ for heads and $2/3$ for tail. A stubborn person tosses this coin until it lands heads-up.
  (a) Describe the sample space.
  (b) Find the probability it takes exactly 3 tosses.
  (c) Find the probability that three or more tosses are necessary.